THEORY OF DYNAMIC MEASUREMENTS ON A TORSIONAL PENDULUM OF HIGH QUALITY FACTOR

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There are certain difficulties in making precision measurement in which the detecting component is a low-frequency mechanical oscillator. In the measurement of low-frequency inputs, one can weaken the elastic coupling in the oscillator and increase the inertia in order to reduce the natural frequency, but this leads firstly to increased complexity in the internal links, which hinders obtaining an explicit analytic relationship between the input and the deflection, and secondly to additional dynamic distortions. Here the traditional theory enables one to establish only an indirect relationship between the observed quantities (the deflection from the equilibrium position, the phase, and the frequency or period) and the input signal, which in practice makes it necessary to generate fairly large volumes of data in order to identify the measured quantity. A typical example is that of a torsional pendulum, which is used in geophysics applications [1] as well as in fundamental research (checking the principle of equivalence, or determining the gravitational constant [1]).

The unsymmetrical torsional pendulum shown in Fig. 1 consists of the thin torsional filament 1 having torsional rigidity K, to the lower end of which is attached the connecting rod 2 of length l working with the arm 3, which has moments of inertia J_x, J_y, J_z, and J_{xz}. A system of equations [3] in the angular variables describes the behavior of this unsymmetrical torsional pendulum: θ_1, θ_2, ψ_1, ψ_2, the angles formed in the ZOY and ZOX planes by the projections of the filament and the connecting rod with the OZ vertical azis, and φ the angle of filament torsion. The external perturbations acting on the pendulum include the second derivatives of the Earth's gravitational potential W_{xx}, W_{yy}, W_{xz} and the accelerations to the point of support due to seismic factors $\ddot{x}$ and $\ddot{y}$. The case $J_{xz} = 0$ corresponds to the common torsional pendulum with a horizontal arm.

The external signal acting on the torsional pendulum is recorded from the angle of rotation (in the torsional degree of freedom φ) or from the period of the torsional oscillations T. As the accuracy in angular measurements is comparatively low, it is preferable to measure the variations in the period. However, although time-interval measurement is more accurate, this accuracy is not realized with the torsional pendulum because there are additional pendulum oscillations produced by microseisms, while the current theory of torsional pendulums does not give direct analytical relationships between the period of the torsional oscillations or the variations in these and the external inputs. This means that one cannot determine the input characteristics from the measurements or analyze the errors in the dynamic measurement method.

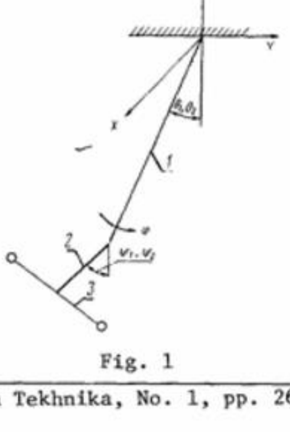

Fig. 1

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The difficulties are eliminated to a certain extent if we transfer from the set of traditional dynamic variables (angle of rotation and angular velocity) to the action variable (phase), since a torsional pendulum working under dynamic conditions with preliminary excitation of the torsional motion is an example of an adiabatically invariant measurement system [4, 5], for which the action is $I(t) = I_0 + \Delta I(t)$, where $\Delta I(t) \ll I_0$, $I_0 = \text{const}$.

Then the equation of motion [3] for the information-bearing torsional degree of freedom is as follows in the action-phase variables:

$$\dot{I} = \frac{\dot{f}(t)}{\Omega_0^2}\sqrt{2J_z\Omega_0 I}\sin\Phi;$$
$$\dot{\Phi} = \Omega_0 + \frac{\dot{f}(t)}{\Omega_0^2}\sqrt{\frac{J_z(\Omega_0)}{2I}}\cos\Phi,$$

where $\Omega_0^2 = K/J_z$, while the generalized force for the infralow-frequency seismic input and the second derivatives of the gravitational potential takes the form

$$f(t) = \frac{J_{xz}\Omega_0^2\ddot{y}(t)}{J_z\omega^2 l} - \frac{J_{xz}x^{IV}(t)}{J_z g}\varphi_0\cos\Omega_0 t + \left[\frac{J_x - J_y}{J_z}(W_{xx} - W_{yy}) - \frac{J_{xz}}{J_z}W_{xz}\right]\varphi_0\cos\Omega_0 t$$

($\omega^2 = g/l$, g is the acceleration due to gravity, and φ_0 is the angle of preliminary deflection of the pendulum in the torsional degree of freedom).

The period T(t) of the torsional oscillations is given by

$$2\pi = \int_t^{t+T(t)} \dot{\Phi}(\tau)\,d\tau.$$

As $I \sim I_0 = \text{const}$ in measurements with a dynamic adiabatically invariant system, it is possible to construct approximate solutions having high accuracy and compact analytic form. With this torsional pendulum, the current value of the torsional-oscillation period is related to the seismic and gravitational inputs by

$$T(t) = T_0 + \frac{J_{xz}}{J_z g\varphi_0\Omega_0}\int_t^{t+T_0} y^{III}(\tau)\sin\Omega_0\tau\,d\tau -$$
$$- \frac{J_{xz}}{2J_z g\Omega_0}\int_t^{t+T_0} x^{III}(\tau)\sin 2\Omega_0\tau\,d\tau - [(J_x - J_y)(W_{xx} - W_{yy}) - J_{xz}W_{xz}]\frac{\pi}{J_z\Omega_0^3}, \quad (1)$$

where $T_0 = 2\pi/\Omega_0$ is the period of the free torsional oscillations.

At present, the seismic noise level restricts the accuracy of gravitational research and measurements on the gravitational constant, so (1) enables one to correct for the effects of seismic oscillations. Also, (1) shows that one can use an unsymmetrical torsional pendulum as a seismic detector in order to record an external seismic signal with frequencies $\Omega_0 < \omega_c < 2\Omega_0$ at the level representing the limit attainable in current seismometry.

A study has been made of the errors in the dynamic method of measuring gravitational inputs via (1), which shows that the largest error is the static or instrumental error [6], which is related to the restricted accuracy in determining the moments of inertia of the arm δJ_x, δJ_y, δJ_z, δJ_{xz}, which is due to errors in manufacture, installation, and adjustment, and in addition there is the extra dynamic error δT_c due to the low-frequency seismic noise, which is difficult to filter out by spectral-analysis methods. The parameters actually attainable in this system are such that the optical and electronic parts of the recording system and the thermal noise give rise to errors that are no more than a fraction of a percent of the main errors.

The overall absolute error is given by

$$\delta T=[(\delta J_x+\delta J_y+\frac{|J_x-J_y|}{J_z}\delta J_z)|W_{xx}-W_{yy}|+$$
$$+(\delta J_{xz}+\frac{|J_{xz}|}{J_z}\delta J_z)|W_{xz}|]\frac{\pi}{J_z\Omega_0^3}+\delta T_c.$$

We estimated the error for an unsymmetrical torsional pendulum having the parameters $J_x = J_z = J_{xz} = J_y/2 = 10^{-4}$ kg·m^2, T_0 = 1800 sec, and φ_0 = 0.05 rad acted on by seismic noise with amplitude $A_x = A_y = 10^{-5}$ m and periods T_c = 10-100 sec, which gave $\delta T_c = 2.5 \cdot 10^{-3}$ sec.

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