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GENERAL PROBLEMS IN THE THEORY OF TRANSFER

# TEMPERATURE FLUCTUATIONS OF MOLECULAR AND PHOTON GASES IN A CYLINDRICAL TUBE OF SMALL RADIUS

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*A study has been made of equilibrium temperature fluctuations of molecular and photon gases in a cavity that is bounded by a cylindrical surface and is in a condensed medium with a high thermal conductivity. It has been shown that the indicated fluctuations belong to the class of non-Markovian random processes and require stochastic integral equations for their description. Basic statistical characteristics of the change in the cavity temperature, including characteristic functions and spectral densities of fluctuation power, have been determined.*

***Keywords:*** *medium with a microstructure, thermal conductivity, non-Markovian process, integral stochastic equation.*

Irreversible processes in a medium with a microstructure are accompanied by random variations in their parameters and give rise to hereditary properties of the medium. The indicated variations may be due to thermodynamic flows in the medium, random variations in its kinetic coefficients, and other factors.

The necessity of taking account of the hereditary properties of a medium with a microstructure limits the application, to it, of a traditional method of description of random irreversible processes in condensed media. The cited method is based on the well-developed theory of differential stochastic systems and reduces the process in question to a Markovian form [1]. Instead of this approach, one can employ the method of integral stochastic operators, which takes account of the medium's heredity. The irreversible process itself becomes a non-Markovian random process in such description.

We emphasize that a study of even the simplest irreversible processes is made with linear integral equations and non-Markovian functions characterizing the change in the quantities in question. In particular, such phenomena include the motion of a spherical Brownian particle in a viscous medium [3], heat conduction and diffusion in the medium around a spherical or cylindrical body [4, 5], Becquerel-type luminescence [6], and other processes.

We note that integral transforms enjoy wide use in describing thermal conductivity. They are applied in [7] to finding analytical solutions of boundary-value problems in an infinite domain bounded by a cylindrical surface. A method of solution of nonlinear nonstationaryheat-conduction problems with nonlinear Volterra integral equations of the second kind is proposed in [8].

The present work seeks to investigate temperature fluctuations of the gas (molecular or photon) in a small-radius tube bounded by a cylindrical surface around which there is s medium with a high thermal conductivity. Note that a study of the gas in a spherical microcavity surrounded by a medium with the indicated properties has been made in [9].

**Formulation of the Problem.** Consideration is given to an infinitely long straight cylindrical tube of small radius $R$ surrounded by a continuous unbounded medium (liquid or a solid) with a high thermal conductivity $\kappa$ and a temperature diffusivity $\chi$ (Fig. 1). Inside the tube, there is equilibrium thermal radiation (photon gas) at the temperature $T(t)$ and a saturated steam (if the medium in a liquid phase) characterized by the thermal conductivity $\kappa_v$, the thermal diffusivity $\chi_v$, and the pressure $p(T)$ dependent on the temperature $T(t)$ of the steam inside the tube.

Transfer of thermal energy through the tube boundary to the environment results from the constant evaporation of liquid particles into the volume of the cylindrical tube, condensation of the steam, and radiation of phonons and their absorption. In the case of equilibrium the mean value of the heat flux $q_T(t)$ related to such transfer is equal to zero during a fairly long lime. However, the flux $q_T(t)$ experiences constant fluctuations due to the random change in the number of condensed (evaporated) gas particles and photons.

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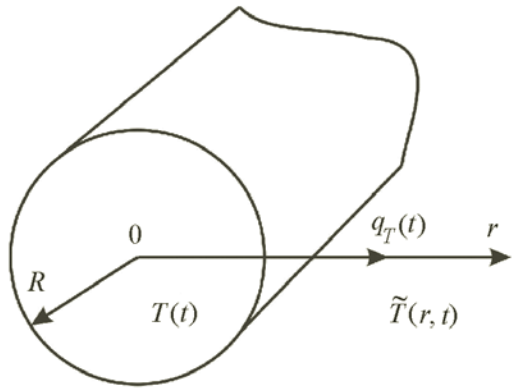



Fig. 1.Cylindrical tube with a gas in a heat-conducting medium.

We will assume that the medium′s temperature $\tilde{T}(r, t)$ outside the tube is only dependent on the time $t$ and the distance $r$ to the cylinder axis, and the gas temperature inside the cylinder is everywhere identical and equal to the temperature of its surface: $T(t) = \tilde{T}(R, t)$. Such assumptions are true of the considered case of the high thermal conductivity of the medium, the tube of a small radius, and $r/R$ values moderately higher than unity.

For the function $\tilde{T}(r, t)$, we have the heat-conduction equation

$$\frac{\partial \tilde{T}(r, t)}{\partial t} = \chi \left( \frac{\partial^2 \tilde{T}(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{T}(r, t)}{\partial r} \right) \tag{1}$$

with initial and boundary conditions

$$\tilde{T}(r, t)\Big|_{t=0} = T_0, \quad \tilde{T}(r, t)\Big|_{r=R} = T(t) . \tag{2}$$

The heat flux $q_T(t)$ through the boundary of the tube is determined by the expression

$$q_T(t) = -\frac{1}{2\pi R} \frac{dU(t)}{dt} , \tag{3}$$

where "–" is attributed to the selected positive direction of the flux from the tube into the medium. The internal energy $U(t)$ of the steam particles inside the tube is determined by their total number $N(t)$ (per unit length) and by the temperature $T(t)$

$$U(t) = \frac{3}{2} N(t) k_B T(t) . \tag{4}$$

The number of particles $N(t)$ can be found from the equation of state, which takes on the form

$$p(t) = \frac{N(t)}{V_0} k_B T(t) = \frac{m_0}{\mu V_0} R_0 T(t) \tag{5}$$

for moderately high temperatures.

To determine the dependence $p(T)$ we will assume that the steam in the tube is saturated and will use the Clausius–Clapeyron equation

$$\frac{dp(T)}{dT} = \frac{Q_{rel}}{V_m T} . \tag{6}$$

Expressing the medium′s specific volume in (5) as $V_m = V_0/m_0$ and substituting this expression into (6), we obtain, after the integration, an expression for the dependence of the pressure of the saturated steam on temperature:

$$p(T) = p_0 \exp\left[\frac{Q_{\rm rel}\mu}{R_0}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]. \quad (7)$$

With this formula, we obtain, from (4) and (5), the following relation for the internal energy $U(t)$ of the molecular steam in the volume the tube of a unit length:

$$U(t) = \frac{3}{2} p_0 V_0 \exp\left[\frac{Q_{\rm rel}\mu}{R_0}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right].$$

Then the heat flux $q_T(t)$ related to the molecular transfer of gas particles through the boundary of the tube will be determined by the relation

$$q_T(t) = -\frac{3}{4}\frac{Q_{\rm v}\mu p_0 R}{R_0 T^2} \exp\left[\frac{Q_{\rm rel}\mu}{R_0}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]\frac{dT(t)}{dt}.$$

Assuming that the variations in the gas temperature in the tube are small ($|T(t) - T_0| << T_0$), we reduce the last expression to the form

$$q_T(t) = -\frac{3}{4}\frac{Q_{\rm rel}\mu p_0 R}{R_0 T^2}\frac{dT(t)}{dt}. \quad (8)$$

An analogous formula for the heat flux $q_T(t)$ due to the emission and absorption of photons can be obtained using the relation for the internal energy of the equilibrium thermal radiation in the volume $V_0$:

$$U(t) = \frac{4\sigma}{c} T^4(t) V_0 .$$

Hence, for the indicated flux, we find

$$q_T(t) = -\frac{8\sigma R T_0^3}{c}\frac{dT(t)}{dt}. \quad (9)$$

It is easily seen from the obtained expressions (8) and (9) that the heat flux $q_T(t)$ related to the transfer of the steam particles and the transfer of photons alike can be represented by the formula of the general form

$$q_T(t) = -\frac{1}{A}\frac{dT(t)}{dt}, \quad (10)$$

in which the quantity $A$ is dependent on the problem′s parameters and is equal to $A = \frac{4}{3}\frac{R_0 T_0^2}{Q_{\rm rel}\mu p_0 R}$ for the case of the heat transfer of energy by steam particles and to $A = \frac{c}{8\sigma R T_0^3}$ for the case of the transfer of energy by photons.

**Stochastic Integral Equation.** As was noted earlier, the heat flux $q_T(t)$ contains, in addition to the found component related to the temperature of the molecular or photons gas in the tube, the random term $\xi_{q_T}(t)$ related to the fluctuations of the number of gas particles and of the medium′s temperature. The character of the statistical properties of this term depends on the particular conditions of the problem. In most cases of major importance the random component $\xi_{q_T}(t)$ of the heat flux $q_T(t)$ through the boundary of the tube may be assumed to be white noise with intensity $\nu$ and mean value $\langle\xi_{q_T}(t)\rangle = 0$. Needless to say, the spectrum of such noise is bounded by a certain frequency $\omega_{\max}$ to which there corresponds the characteristic relaxation time $\tau_{\rm r} = 1/\omega_{\max}$ evaluated as

$$\tau_{\rm r} = \frac{R^2}{\chi_v} \quad (11)$$

for the fluctuations of the heat flux related to the transfer of steam particles, or as

$$\tau_{\rm r} = \frac{R}{c}, \quad (12)$$

if the heat-flux fluctuations are due to the radiation and absorption of photons.

The intensity ν of random variations in the heat flux $\xi_{qT}(t)$ can also be evaluated with the problem′s parameters [10]. For molecular and photon gases, this parameter is determined by the relations

$$\nu = \frac{\kappa_{\nu}}{R^3} k_{\rm B} T_0^2 \ , \tag{13}$$

$$\nu = \frac{\sigma}{R^2} k_{\rm B} T_0^5 \ . \tag{14}$$

Let us evaluate the quantities prescribed by relations (11)–(14) for the case of a cylindrical tube of radius $R = 1$ μm inside which there is a saturated steam ($\kappa_{\rm v} = 0.02$ W/(K·m) and $\chi_{\rm v} = 3.5\cdot10^{-4}$ m$^2$/s) at the temperature $T_0 = 300$ K. For the characteristic time $\tau_{\rm r}$ and the intensity of fluctuations ν of the heat flux, we respectively obtain $\tau_{\rm r} = 2.9\cdot10^{-9}$ s and $\nu = 0.025$ J$^2$/(m$^4$·s). Analogous evaluations for the photon gas yield $\tau_{\rm r} = 3\cdot10^{-15}$ s and $\nu = 1.9\cdot10^{-6}$ J$^2$/(m$^4$·s).The found values of intensity show that, in the presence of the saturated molecular steam, the contribution of the component due to the presence of equilibrium thermal radiation to the fluctuations of the heat flux is small. This fact enables us to disregard the photon part of random variations in the heat flux given the molecular part.

Setting the sum of the flux $q_T(t)$ prescribed by expression (10) and of the random flux $\xi_{qT}(t)$ equal to the general relation for the heat flux through the boundary of the tube

$$q_T(t) = -\kappa \left.\frac{\partial \tilde{T}(r, t)}{\partial r}\right|_{r=R} ,$$

we obtain

$$-\kappa \left.\frac{\partial \tilde{T}(r, t)}{\partial r}\right|_{r=R} = -\frac{1}{A}\frac{dT(t)}{dt} - \xi_{qT}(t) \ . \tag{15}$$

Here the sign before the random flux $\xi_{qT}(t)$ is determined by the positive direction of the flux selected earlier.

It may be shown [11] that if the heat $Q(t)$ is released on each unit length of the considered cylindrical surface of the tube at a certain instant of time τ, the medium′s temperature at $r > R$ at the instant of time $t$ ($t > \tau$) is prescribed by the relation

$$\tilde{T}(r, t) = \frac{\chi}{\kappa} Q(\tau) G(r, R, t-\tau) ,$$

in which Green′s function $G(r, R, t - \tau)$ is defined as

$$G(r, R, t-\tau) = \frac{1}{4\pi\chi(t-\tau)} \exp\left(-\frac{r^2 + R^2}{4\chi(t-\tau)}\right) I_0\left(\frac{Rr}{2\chi(t-\tau)}\right) .$$

Using the fact that $Q(\tau)\delta(t-\tau) = 2\pi R q_T(t)$, for the medium′s temperature $\tilde{T}(r, t)$ at the distance $r$ from the axis of the tube for the case of an arbitrary heat flux $q_T(t)$ through its surface, we obtain

$$\tilde{T}(r, t) = \frac{R}{2\kappa}\int_0^t \frac{q_T(\tau)}{t-\tau} \exp\left(-\frac{r^2 + R^2}{4\chi(t-\tau)}\right) I_0\left(\frac{Rr}{2\chi(t-\tau)}\right) d\tau \ . \tag{16}$$

We find the derivative $\frac{\partial \tilde{T}(r, t)}{\partial r}$ from expression (16), using the equality $\frac{\partial I_0(z)}{\partial z} = I_1(z)$ [12]:

$$\frac{\partial \tilde{T}(r, t)}{\partial r}$$

$$= \frac{R}{4\chi\kappa}\int_0^t \frac{q_T(\tau)}{(t-\tau)^2} \exp\left(-\frac{r^2 + R^2}{4\chi(t-\tau)}\right)\left[R I_1\left(\frac{Rr}{2\chi(t-\tau)}\right) - r I_0\left(\frac{Rr}{2\chi(t-\tau)}\right)\right] d\tau \ .$$

The derivative found, after its substitution into relation (15) and with account of (10), leads to aVolterra integral equation of the second kind

$$Z(t) + \int_0^t K_0(t-\tau)\, Z(\tau)\, d\tau = -A\xi_{q_T}(t)\,, \tag{17}$$

in which we have introduced the following notation for the rate of change in the gas temperature in the tube and the kernel of the integral equation $K_0(t)$:

$$Z(t) = \frac{dT(t)}{dt}\,,$$

$$K_0(t-\tau) = \frac{R^2}{4\chi(t-\tau)^2}\exp\left(-\frac{R^2}{2\chi(t-\tau)}\right)\left[I_1\left(\frac{R^2}{2\chi(t-\tau)}\right) - I_0\left(\frac{R^2}{2\chi(t-\tau)}\right)\right]. \tag{18}$$

In connection with the fact that the propagation of heat in the space around the cylindrical tube with the molecular or photon gas in it is described by the integral equation (17) not reducible to a finite system of differential equations, fluctuations of the quantity $Z(t)$, and hence the quantities $T(t)$ and $q_T(t)$, may be classified as random non-Markovian processes. This means that random variations in the indicated quantities possess hereditary properties: their statistical characteristics, beginning with a certain time $\tau$, are dependent on the behavior of these functions at $t < \tau$.

**Solution of the Integral Stochastic Equation.** The considered case of a gas-containing tube of small radius (the condition $\frac{R^2}{\chi} << \delta t$, where $\delta t$ is the characteristic time corresponding to the time of free motion of molecular-gas particles or the time of motion of photons on a length equal to the distance between neighboring particles in a photon gas) enables us to expand the exponents and Bessel functions in (18) in a series with retention of just the first two terms. A supplementary physical condition $t - \tau > \delta t$ for the kernel $K_0(t - \tau)$ enables us to write the expression

$$K_0(t-\tau) = -\frac{R^2}{4\chi(t-\tau)^2}\,.$$

Then the initial integral equation (17) is reduced to the form

$$Z(t) - \frac{R^2}{4\chi}\int_0^t \frac{1}{(t-\tau)^2}\, Z(\tau)\, d\tau = -A\xi_{q_T}(t)\,. \tag{19}$$

Equation (19) may be written with its resolvent:

$$Z(t) = A\xi_{q_T}(t) + A\int_0^t F(t-\tau)\xi_{q_T}(\tau)\, d\tau\,, \tag{20}$$

where the function $F(t - \tau)$ is determined by the recurrence relation

$$F(t-\tau) = \sum_{n=1}^{\infty} F_n(t-\tau)\,, \tag{21}$$

in which

$$F_1(t-\tau) = \frac{R^2}{4\chi}\frac{1}{(t-\tau)^2}\,, \quad F_n(t-\tau) = \int_{\tau+\delta t}^{t-\delta t} F_1(t-s)F_{n-1}(s-\tau)\, ds, \quad n > 1\,.$$

The above condition of a small radius makes series (21) fast convergent. The inequality $\frac{R^2}{\chi} << \delta t$ leaves just the first term of series (21). Then the solution prescribed by Eq. (20) is written in the form

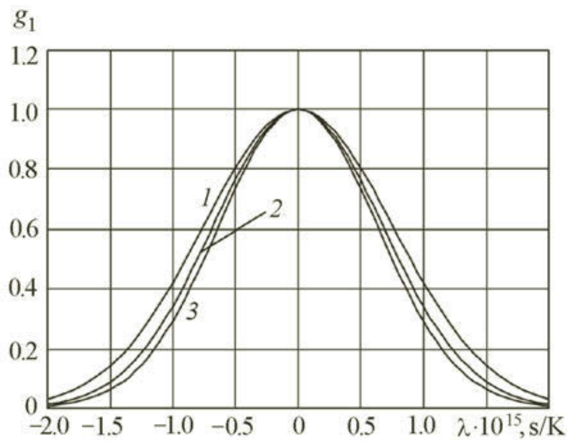



Fig. 2. Plots of the function $g_1(\lambda, t)$ prescribed by Eq. (23): 1) $t = 1.5\delta t$; 2) $t = 2.0\delta t$; 3) $t = \infty$.

$$Z(t) = A\xi_{qT}(t) + \frac{AR^2}{4\chi}\int_0^t \frac{1}{(t-\tau)^2}\,\xi_{qT}(\tau)\,d\tau\,. \tag{22}$$

**Statistical Characteristics.** The found integral stochastic equation (22) enables us to find any statistical characteristics of the process $Z(t)$ using the method developed in [13] for description of non-Markovian processes that are prescribed by linear integral transforms. Taking into account that the random heat flux $\xi_{qT}(t)$ is white noise with an intensity ν, for the one-dimensional $g_1(\lambda, t)$ and multidimensional $g_L(\lambda_1, \ldots, \lambda_L, t_1, \ldots, t_L)$ characteristicfunctions we obtain

$$g_1(\lambda, t) = \exp\left[-\frac{1}{96}\frac{\nu A^2 R^4 \lambda^2}{\chi^2}\left(\frac{1}{\delta t^3} - \frac{1}{t^3}\right)\right], \tag{23}$$

$$g_L(\lambda_1, \ldots, \lambda_L, t_1, \ldots, t_L)$$
$$= \exp\left[-\frac{1}{48}\frac{\nu A^2 R^4}{\chi^2}\sum_{\substack{k,l=1,\\ k<l}}^{L}\frac{\lambda_k\lambda_l}{(t_l - t_k)^2}\left(\frac{1}{\delta t} + \frac{t_k^2 - t_l^2 + t_k t_l}{t_k t_l (t_l - t_k)} + \frac{2}{t_l - t_k}\ln\frac{t_l \delta t}{t_k (t_l - t_k)}\right)\right]. \tag{24}$$

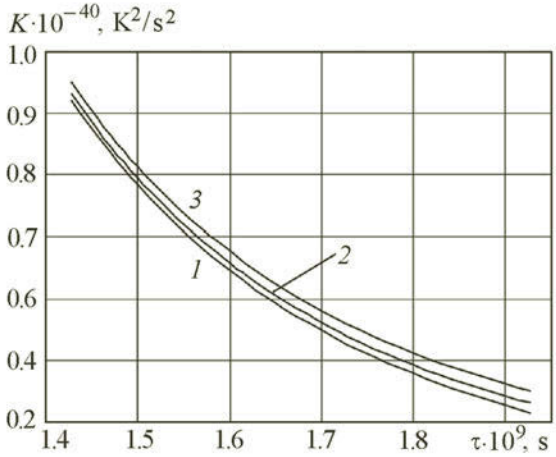
The functions $g_1(\lambda, t)$ prescribed by Eq. (23) are plotted in Fig. 2 for different values of $t$. Here and in what follows we use, as an example, cylindrical tubes of a small radius placed in water ($R = 10^{-6}$ m, $\nu = 2.5\cdot 10^4$ J$^2$/(m$^4$·s), $\chi = 2\cdot 10^{-7}$ m$^2$/s, and $A = 2.1\cdot 10^6$ K·m$^2$/J).

The characteristic functions (23) and (24) make it possible to determine any statistical characteristics of the process $Z(t)$. In particular, the mathematical expectation $\langle Z(t)\rangle$ and the variance $D_Z(t)$ are prescribed by the equalities

$$\langle Z(t)\rangle = \left.\frac{\partial g_1(\lambda, t)}{i\partial\lambda}\right|_{\lambda=0} = 0\,,$$

$$D_Z(t) = \langle Z^2(t)\rangle = -\left.\frac{\partial^2 g_1(\lambda, t)}{\partial\lambda^2}\right|_{\lambda=0} = \frac{1}{48}\frac{\nu A^2 R^4}{\chi^2}\left(\frac{1}{\delta t^3} - \frac{1}{t^3}\right).$$

To the steady-state heat-conduction process for which $t \to \infty$, there corresponds the stationary value of the variance of the rate of change in the gas temperature in the tube:



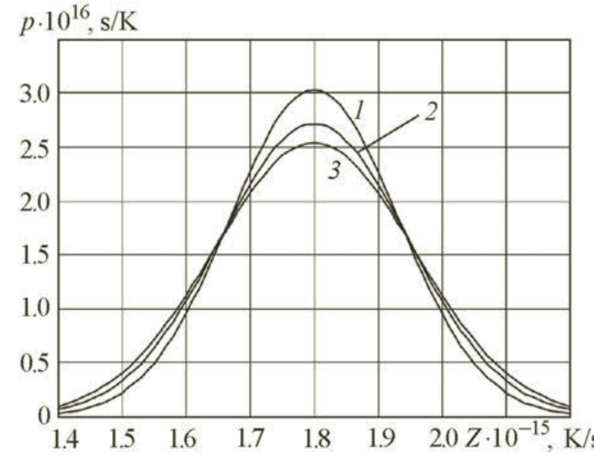




Fig. 3. Plots of the function $K(t-\tau, t)$ prescribed by Eq. (26). The values of the time $t$ are the same as in Fig. 2.

Fig. 4. Plots of the probability density function $p(Z, t)$ prescribed by Eq. (28). The values of time $t$ are the same as in Fig. 2.

$$D_Z(t)\Big|_{t\to\infty} = \frac{1}{48}\,\frac{\nu A^2 R^4}{\chi^2 \delta t^3}\,. \tag{25}$$

The correlation function $K(t_1, t_2) = \langle Z(t_1)Z(t_2)\rangle$ may be obtained from the two-dimensional characteristic function $g_2(\lambda_1, \lambda_2; t_1, t_2)$ using formula (24) at $L = 2$:

$$K(t_1, t_2) = -\left.\frac{\partial^2 g_2(\lambda_1, \lambda_2; t_1, t_2)}{\partial\lambda_1\partial\lambda_2}\right|_{\substack{\lambda_1=0,\\ \lambda_2=0}}$$
$$= \frac{1}{96}\,\frac{\nu A^2 R^4}{\chi^2}\,\frac{1}{(t_2-t_1)^2}\left[\frac{1}{\delta t} + \frac{t_1^2 - t_2^2 + t_1 t_2}{t_1 t_2 (t_2 - t_1)} + \frac{2}{t_2 - t_1}\ln\frac{t_2 \delta t}{t_1(t_2 - t_1)}\right]. \tag{26}$$

In particular, to the steady-state process ($t_1 \to \infty$, $t_2 \to \infty$), there corresponds the stationary correlation function $K(t_1, t_2) = K(t_2 - t_1) = K(\tau)$ of the form

$$K(\tau) = \frac{1}{96}\,\frac{\nu A^2 R^4}{\chi^2 \tau^2}\left(\frac{1}{\delta t} + \frac{1}{\tau} + \frac{1}{\tau}\ln\frac{\delta t}{\tau}\right). \tag{27}$$

At $\tau = \delta t$, formula (27) yields expression (25). The functions $K(t-\tau, t)$ prescribed by relation (26) are plotted in Fig. 3 for different values of the parameter $z$. Relation (26) for the correlation function $K(t_1, t_2)$ enables us to find one-sided spectral density of the process $Z(t)$ using the definition

$$G_Z(\omega, t) = 2\int_0^t K(t-\tau, t)\cos\omega\tau d\tau\,.$$

For the one-dimensional probability density function $p(Z, t)$, we obtain the expression

$$p(Z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g_1(\lambda, t) \exp(-i\lambda Z)\, d\lambda$$
$$= \sqrt{\frac{1}{2\pi D_Z(t)}} \exp\left(-\frac{Z^2}{2D_Z(t)}\right). \quad (28)$$

Figure 4 depicts dependence (28) for different values of $t$. It is easily seen that the plot of the probability density of fluctuations of the rate of change in the gas temperature in the tube $Z(t)$ becomes "smeared" along the $Z$ axis with time, tending to a stationary Gaussian curve at $t \to \infty$.

**Conclusions.** Description of fluctuations of physical quantities characterizing the process of heat conduction in a medium containing microtubes requires integral stochastic equations, and the fluctuations themselves are non-Markovian random processes. The obtained results are of importance in describing random temperature fluctuations in media with a microstructure.

## NOTATION

$c$, velocity of sound in vacuum, m·s$^{-1}$; $D_Z(t)$, variance of the rate of change in the gas temperature in a tube, K$^2$·s$^{-2}$; $G(r, R, t)$, Green's function, m$^{-2}$; $I_0(x)$ and $I_1(z)$, modified Bessel functions of zero and first orders; $i$, imaginary unit; $K(t_1, t_2)$ and $K(\tau)$, correlation and stationary correlation functions of the rate of change in the gas temperature in a tube, K$^2$·s$^{-2}$; $k_B$, Boltzmann constant, J·K$^{-1}$; $m_0$, mass of the gas in the volume of a tube of unit length, kg; $N(t)$, number of particles in a tube per its unit length, m$^{-1}$; $p(T)$ and $p_0$, pressures of the saturated steam in a tube at the temperatures $T$ and $T_0$, Pa; $p(Z, t)$, one-dimensional probability density function for the rate of change in the gas temperature in a tube, K$^{-1}$·s; $Q(\tau)$, heat instantaneously released on a unit length of the cylindrical surface of a tube, J·m$^{-1}$; $Q_{rel}$, specific heat of vaporization of the liquid surrounding the tube, J·kg$^{-1}$; $q_T(t)$, heat flux through the tube surface, J·m$^{-2}$·s$^{-1}$; $R$, radius of a cylindrical tube, m; $R_0$, gas constant, J·mole$^{-1}$·K$^{-1}$; $r$, distance from the rube axis, m; $T(t)$, gas temperature inside the tube, K; $T_0$, initial gas temperature in a tube, K; $\tilde{T}(r, t)$, temperature of the medium surrounding the tube, K; $t$, time, s; $U(t)$, internal energy of the gas in a tube per its unit length, J·m$^{-1}$; $V_0$, volume of a tube of unit length, m$^{-3}$; $V_m$, specific volume (volume of a unit mass), m$^3$; $Z(t)$, rate of change in the gas temperature in a tube, K·s$^{-1}$; $\delta t$, characteristic time corresponding to the time of free motion of a molecular gas or to the time of motion of photons on a length equal to the distance between neighboring particles in a photon gas, s; $\kappa$ and $\kappa_v$, thermal-conductivity coefficients of the medium surrounding the tube and of the saturated steam in the tube, J·m$^{-1}$·s$^{-1}$; $\lambda$, parameter of the characteristic function, s/K; $\mu$, molecular mass of the gas in a tube, kg·mole$^{-1}$; $\nu$, intensity of the random heat flux through the tube surface, J$^2$·m$^{-4}$·s$^{-1}$; $\xi_{qT}(t)$, random heat flux through the tube surface, J·m$^{-2}$·s$^{-1}$; $\sigma$, Stefan–Boltzmann constant, J·K$^{-4}$·m$^{-2}$·s$^{-1}$; $\tau$ and $\tau_r$, time and characteristic time of relaxation of fluctuations of the heat flux through the tube surface, s; $\chi$ and $\chi_v$, thermal-diffusivity coefficient of the medium surrounding the tube and of the saturated steam in the tube, m$^2$·s$^{-1}$; $\omega$ and $\omega_{max}$, frequency and frequency bounding the spectrum of power of the heat flux through the tube surface, s$^{-1}$. Subscripts: r, relaxation; rel, relative; v, vapor, steam.

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