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# The high-frequency gravitational waves in exact inflationary models with Gauss-Bonnet term

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**Abstract.** We consider the exact inflationary solutions for a single scalar field with an arbitrary potential and an arbitrary nonminimal coupling to the Gauss-Bonnet term in the flat Friedmann-Robertson-Walker universe on the basis of connection with standard inflation. The possibility of the registration of relic gravitational waves by using the phenomenon of a low-frequency optical resonance in Fabri-Perot interferometers are also received.

## 1. Introduction

Inflation in the early universe has become the standard model for the generation of cosmological perturbations in the universe, the seeds for large-scale structure and temperature anisotropies of the cosmic microwave background. The simplest scenario of cosmological inflation is based upon a single, minimally coupled scalar field with a potential of self-interaction. Quantum fluctuations of this inflaton field give rise to an almost scale-invariant power spectrum of isentropic perturbations [1–4].

Despite the fact that the inflationary scenario solves the problem of the big bang theory, for instance, the horizon and flatness problem, there are still unsolved problems such as initial singularity problem, and quantum gravity.

String theory is often regarded as the leading candidate for unifying gravity with the other fundamental forces and for a quantum theory of gravity. The effective supergravity action from superstrings induces correction terms of higher order in the curvature, which may play a significant role in the early universe. The simplest such correction is the Gauss-Bonnet term in the low-energy effective action of the heterotic string [5].

For the case of four-dimensional Friedmann-Robertson-Walker universe with non-minimal coupling of a scalar field and Gauss-Bonnet scalar, the solutions were obtained in slow-roll approximation [6, 7] and in the case of inflation which is driven by the interaction of a scalar field and Gauss-Bonnet term without the potential [8]. But, in the work [9] it was shown that the Gauss-Bonnet inflation without an inflaton potential is not phenomenologically viable. The some exact solutions for Gauss-Bonnet inflation with potential were received in works [10–12].

The method of exact solution for Gauss-Bonnet inflation on the basis of connection with standard inflation is represented in this work. The method of calculation of the the power spectra and energy density of tensor perturbations is also submitted.

Also, the detection of gravitational wave background (GWB) gives the important information about the early universe, and can be used to test the theoretical models. In this context, it is



important to develop new methods for the detection of gravitational waves. One of the promising methods for increasing the sensitivity of gravitational antennas in the high frequency part of the spectrum is the use of low-frequency optical resonance phenomenon (LOR), whose presence in the Fabry-Perot interferometer was found in the works [13–15].

## 2. The scalar field dynamical equations with the Gauss-Bonnet term

We consider the background dynamical equations for inflation with the Gauss-Bonnet term that is coupled to a scalar field  $\phi$  in the system of units  $8\pi G = c = 1$  in a spatially flat Friedmann-Robertson-Walker universe [6, 7, 10]

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) + 12\xi H^3 \quad (1)$$

$$-2\dot{H} = \dot{\phi}^2 - 4\ddot{\xi}H^2 - 4\dot{\xi}H(2\dot{H} - H^2) \quad (2)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + 12\xi_{,\phi}H^2(\dot{H} + H^2) = 0, \quad (3)$$

where a dot represents a derivative with respect to the cosmic time  $t$ ,  $H \equiv \dot{a}/a$  denotes the Hubble parameter, and  $V_{,\phi} = \partial V/\partial\phi$ ,  $\xi_{,\phi} = \partial\xi/\partial\phi$ . Since  $\xi$  is a function of  $\phi$ ,  $\dot{\xi}$  implies  $\dot{\xi} = \xi_{,\phi}\dot{\phi}$ .

The equation (3) is the result of the equations (1–2), therefore, we will consider the equations (1–2) as completely describe the dynamics of the scalar field in the case of Gauss-Bonnet inflation.

## 3. The method of exact solutions

Now, we define the connection between standard inflation and inflation with Gauss-Bonnet correction

$$\bar{H} = H(1 - 2\xi H) \quad (4)$$

The equations (1–2), in this case, are written as follows

$$\frac{1}{2}\dot{\phi}^2 + V(\phi) = -3H^2 + 6\bar{H}H \quad (5)$$

$$\frac{1}{2}\dot{\phi}^2 = -\dot{\bar{H}} + \bar{H}H - H^2 \quad (6)$$

In the case of  $\xi = const$  (or  $\bar{H} \rightarrow H$ ) equations (5–6) are reduced to the standard inflation.

For  $H \rightarrow \bar{H}$  we obtain the dynamical equations for standard-like inflation, but with considering the action of the coupling of a scalar field and Gauss-Bonnet term on the background dynamics

$$3\bar{H}^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad (7)$$

$$-2\dot{\bar{H}} = \dot{\phi}^2 \quad (8)$$

Further, we write the equations (5–6) in the following form

$$V(\phi) = -2H^2 + 5\bar{H}H + \dot{\bar{H}} \quad (9)$$

$$\frac{1}{2}\dot{\phi}^2 = -\dot{\bar{H}} + \bar{H}H - H^2 \quad (10)$$

Thus, by selecting the Hubble parameter  $H = H(t)$  and the scalar field  $\phi = \phi(t)$  we will generate the exact solutions of equations (9–10).

We consider the quintessential inflationary model with

$$H(t) = B/t \quad (11)$$

$$\phi(t) = \pm\sqrt{B^2 + A - AB} \ln(t) + \phi_0, \quad (12)$$

where  $A, B$  are the positive constants.

The exact solutions are

$$\bar{H}(t) = A/t \quad (13)$$

$$V(\phi) = (-4B^2 + 7AB - A) \exp(\pm 2(\phi - \phi_0)/C) \quad (14)$$

$$\xi(\phi) = \frac{B - A}{4B^2} \exp(\mp 2(\phi - \phi_0)/C), \quad (15)$$

Thus, we obtain  $\bar{H} = \alpha H$ , where  $\alpha = A/B$ .

It is possible to obtain another exact solutions from equations (9–10), but we are interested in a simple quintessential inflationary scenario in this work.

#### 4. Gravitational waves

During inflation, the quantum fluctuations of a scalar field would create the perturbations of metric. On the basis of connection between Gauss-Bonnet and standard inflation by modified Hubble parameter  $\bar{H}$  we obtain the amplitude of relic gravitational waves or tensor perturbations at the end of the inflation in the selected system of units which is defined as follows [16]

$$h_{GW}^2 = \frac{\bar{H}(t = t_i)}{4\pi^2} \quad (16)$$

The spectral energy density of gravitational waves for each stage [16]

$$\Omega_{GW}^{MD}(f) = \frac{3}{8\pi^2} h_{GW}^2 \Omega_{0m} \left( \frac{f_0}{f} \right), f_0 \leq f < f_{MD} \quad (17)$$

$$\Omega_{GW}^{RD}(f) = \frac{1}{6\pi} h_{GW}^2 \Omega_{0r}, f_{MD} \leq f < f_{RD} \quad (18)$$

$$\Omega_{GW}^{kin}(f) = \frac{3}{8\pi^2} h_{GW}^2 \Omega_{0m} \left( \frac{f}{f_{RD}} \right), f_{RD} \leq f < f_{kin}, \quad (19)$$

where  $f_0, f_{kin}, f_{RD}, f_{MD}$  – the frequencies of the gravitational waves at each stage of evolution of the Universe

$$f_0 = \frac{\bar{H}_0}{2} \quad (20)$$

$$f_{MD} = \frac{3}{2\pi} f_0 \left( \frac{\Omega_{0m}}{\Omega_{0r}} \right)^{1/2} \quad (21)$$

$$f_{RD} = \frac{1}{4} f_0 \left( \frac{\Omega_{0m}}{\Omega_{0r}} \right)^{1/2} \frac{T_{rh}}{T_{MD}} \quad (22)$$

$$f_{kin} = \bar{H}_{kin} \left( \frac{T_0}{T_{rh}} \right) \left( \frac{\bar{H}_{rh}}{\bar{H}_{kin}} \right)^{1/3} \quad (23)$$

Here  $\bar{H}_0 = 67.8 \pm 0.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $\Omega_{0m} = 0.308 \pm 0.012$  and  $\Omega_{0r} = (9.230 \pm 0.022) \times 10^{-5}$  are Hubble parameter, the density of matter and radiation at the modern era [17],  $T_{rh} = 1 \times 10^{14} \text{ GeV}$  and  $H_{rh}$  – reheating temperature and Hubble parameter, that we take approximately same as the temperature and Hubble parameter at the end of inflation. The expression (19) corresponds to the high-frequency part of the gravitational waves spectrum.

### 5. The possibility of experimental detection of relic gravitational waves

To date, several projects of searching for gravitational waves, such as projects LIGO (USA), VIRGO (Italy, France), TAMA-300 (Japan), GEO-600 (Germany) and others, are realized. Their main feature is the registration attempt of short bursts of gravitational waves from astrophysical origin rather rare events such as the collapse of stars to black holes or neutron stars merge. To register the events developed detectors having a peak response in the frequency range from 100 to 1000 Hz [18, 19].

An important role in the gravitational-wave experiments plays the detection of gravitational wave background. One of the promising methods for increasing the sensitivity of gravitational antennas in the high frequency part of the spectrum is the use of low-frequency optical resonance phenomenon (LOR), whose presence in the Fabry-Perot interferometer found in the works [13–15]. Preliminary analysis of the sensitivity of the Fabry-Perot interferometer in the high frequency part of the spectrum was made in [20].

In the articles [20, 21], it has been shown that the minimum detectable spectral density of the space-time fluctuations by using low-frequency optical resonance in a Fabry-Perot interferometer can be estimated by the formula

$$G_h(f) > \sqrt{\frac{2\pi\kappa}{c^2T\Delta}} \left( \frac{2\pi\hbar f^{3/2}}{k_e W_0} \right) \quad (24)$$

where  $\kappa$  – phase shift that characterizes the setting of the interferometer,  $c$  – the speed of light,  $T$  – time averaging of the spectral density,  $\Delta$  – loss per cycle of reflections,  $\hbar$  – Planck constant,  $k_e$  – wave number,  $W_0$  – power incident on the Fabry-Perot interferometer monochromatic laser radiation,  $f$  – gravitational wave frequency.

To obtain the value of the energy density of gravitational waves  $\Omega_{GW}(f)$  one can use the formula given in [20]

$$\Omega_{GW}(f) = \frac{4\pi^2}{3H_0^2} f^3 G_h(f) \quad (25)$$

Substituting formula (24) in the expression (25) gives an estimate for the minimum energy density of gravitational waves that can be detected with the use of low-frequency optical resonance

$$\Omega_{GW}(f) > \sqrt{\frac{2\pi\kappa}{c^2T\Delta}} \left( \frac{8\pi^3\hbar f^{9/2}}{3H_0^2 k_e W_0} \right) \quad (26)$$

The values of the frequency and the energy density of relic gravitational waves is limited to conditions [22, 23]:

- The value of the energy density of relic gravitational waves, which can impact on the rate of the primordial nucleosynthesis, shall not exceed

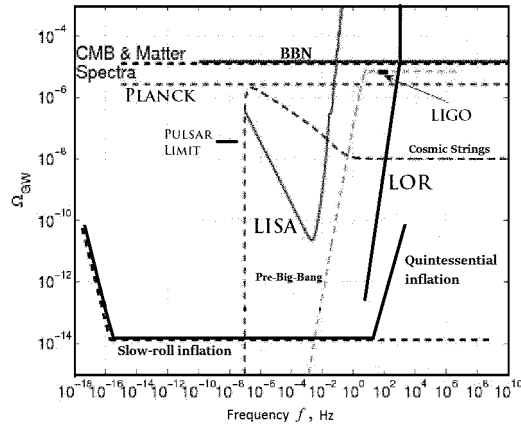
$$\int_{f_0}^{\infty} \Omega_{GW} d \ln f < 1.1 \cdot 10^{-5}, \quad f_0 \approx 10^{-9} Hz. \quad (27)$$

- The temperature value of the scalar field at the stage of inflation  $T_*$  and the frequency of gravitational waves is generated at the horizon scale

$$T_* = 5.85 \cdot 10^6 \left( \frac{f}{Hz} \right) \left( \frac{g_*}{106.75} \right)^{-1/6} GeV \quad (28)$$

$$f = 1.71 \cdot 10^{-7} \left( \frac{T_*}{GeV} \right) \left( \frac{g_*}{106.75} \right)^{1/6} Hz, \quad (29)$$

where  $g_*$  – effective number of degrees of freedom (for standard model  $g_* = 106.75$ ).



**Figure 1.** The GWB spectra produced by the quintessential inflation with potential (14) and slow-roll inflation. Also, the sensitivity of the LOR experiment is compared with other experiments.

## 6. Conclusion

The method of exact solutions for Gauss-Bonnet inflationary models in the flat Friedmann-Robertson-Walker universe was proposed in this work. Also, the technique for calculation of relic gravitational wave's parameters on the basis of connection between standard and Gauss-Bonnet inflation was offered. The way for increasing the sensitivity of gravitational antennas in the high frequency part of the spectrum by using of low-frequency optical resonance phenomenon in Fabry-Perot interferometer was considered as well.

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