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EQUILIBRIUM TEMPERATURE FLUCTUATIONS OF MOLECULAR AND PHOTONIC GASES IN A SPHERICAL MICROCAVITY

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Heat conduction process is investigated in space beyond a spherical microcavity in a solid or liquid in the presence of heat flux fluctuations through the cavity surface. It is demonstrated that random fluctuations of the cavity temperature represent a non-Markov process, and integral stochastic equations are required to describe them. Statistical characteristics of temperature fluctuations, including characteristic functions, density distribution functions, and spectral densities, are obtained. It is demonstrated that the character of small steady-state gas temperature fluctuations in the cavity is independent of the gas properties and its equation of state and is completely determined by the properties of the medium and cavity sizes.

Keywords: heat conduction, medium with a microstructure, non-Markov process, integral transformations.

A description of the heat conduction processes in solids and liquids often calls for a consideration of heat flux and temperature fluctuations caused, for example, by random variations of the power of thermal sources, thermodynamic flows, heat conduction, etc. Especially important is consideration of such fluctuations in media with a microstructure, for example, in solid bodies with cavities or liquids with gas bubbles due to considerable influence of fluctuations of thermodynamic flows on the parameters characterizing the cavities or bubbles including their temperature.

Heat flux and temperature fluctuations in these media can be studied by the standard methods using the differential heat conduction equation complemented by the corresponding stochastic term responsible for a random heat flux. The stochastic differential equation describing the Markov heat conduction process can be solved by the well developed methods of stochastic system theory [1].

However, irreversible physical processes to which the heat conduction phenomenon belongs possess the hereditary properties which in many cases must be taken into account. The character of changes of the parameters describing the medium with a microstructure at a fixed moment of time depends generally on the character of changes of this parameter at the preceding moments of time. Thus, in particular, it is demonstrated that a more exact description of microparticle motion in a viscous medium [2] and of heat conduction and diffusion in the region surrounding a microscopic spherical particle [3] or microfilament [4] is possible only in the context of the theory of non-Markov processes using integral stochastic operators instead of differential ones.

The present work is devoted to a description of equilibrium temperature fluctuations of molecular or photon gas enclosed in a small cavity of a solid body or liquid in the presence of random heat flux through the cavity boundaries. It is demonstrated that this process must be described by integral equations. The statistical characteristics of gas temperature fluctuations in the cavity are determined.

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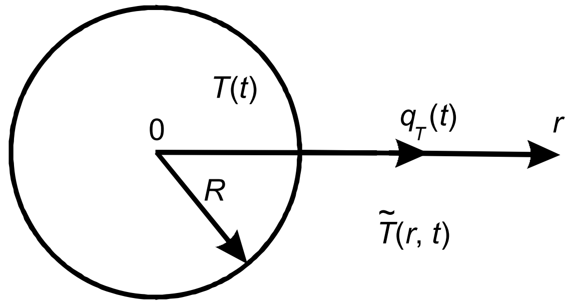

Fig. 1. Cavity with a molecular or photon gas.

PROBLEM FORMULATION

Let us consider a small spherical cavity of constant radius R and volume $V = \frac{4}{3}\pi R^3$ in a liquid or solid body having high heat conductivity κ (Fig. 1). The cavity contains equilibrium thermal radiation of the medium (the photon gas) at the temperature $T(t)$ and in the case of the liquid medium, saturated vapor characterized by thermal diffusivity χ_n and heat conductivity κ_n at the pressure $p(T)$ depending on the temperature $T(t)$ of the external cavity boundary. Possible evaporation of the solid phase is neglected.

Let the initial temperatures of the surrounding medium and vapor (or photon gas) in the cavity be identical and equal to constant temperature T_0. As a result of molecular motion of vapor particles, a portion of them arriving at the interface between the liquid and gas phases can be condensed. The reverse process of liquid particle evaporation into the cavity is also possible. It is obvious that these processes lead to heat transfer through the external cavity boundary (occurrence of the heat flux $q_T(t)$) and hence to changes in the temperature $T(t)$ inside of the cavity. Similarly, photon emission and absorption in the medium also leads to the occurrence of the heat flux $q_T(t)$. Because of the smallness of the cavity and high heat conductivity of the medium, the temperature $T(t)$ in the first approximation can be considered the same within the cavity at any arbitrary moment of time. The temperature of the medium out of the cavity $\tilde{T}(r,t)$ depends, in addition to time, on the distance r to the cavity centre.

For the temperature $\tilde{T}(r,t)$ of the medium out of the cavity, the heat conduction equation assumes the form [5]

$$\frac{\partial \tilde{T}(r,t)}{\partial t} = \frac{\chi}{r} \frac{\partial^2 \left(r\tilde{T}(r,t) \right)}{\partial r^2}, \quad r > R, \tag{1}$$

where χ is the thermal diffusivity of the medium. The initial and boundary conditions for Eq. (1), according to the foregoing, are written as follows:

$$\tilde{T}(r,t)\Big|_{t=0} = T_0, \tag{2}$$

$$\tilde{T}(r,t)\Big|_{r=R} = T(t). \tag{3}$$

The heat flux $q_T(t)$ through the cavity boundary is determined by the expression

$$q_T(t) = -\frac{1}{4\pi R^2}\frac{dU(t)}{dt}, \tag{4}$$

where $U(t)$ is the internal energy of the molecular or photon gas inside of the cavity and the direction outside of the cavity is accepted to be positive.

The internal energy of vapor particles $U(t)$, in addition to the temperature $T(t)$, is determined by the number of particles $N(t)$:

$$U(t) = \frac{3}{2}k_B N(t) T(t), \tag{5}$$

where k_B is the Boltzmann constant. The particle number $N(t)$ can be found from the equation of state for vapor which at not so high temperatures is written in the form

$$p(T) = \frac{N(t)}{V}k_B T(t). \tag{6}$$

To find a temperature dependence of the saturated vapor pressure $p(T)$, we take advantage of the Clapeyron–Clausius equation which in the case under consideration assumes the form

$$dp = \frac{Q_r}{V_m T}dT, \tag{7}$$

where dp and dT are infinitesimal changes of the pressure and gas temperature inside of the cavity, Q_r is the latent heat of vaporization, and V_m is the specific volume. Taking advantage of the ideal gas equation, from Eq. (7) we derive the expression

$$p(T) = p_0 \exp\left[\frac{Q_r \mu}{R_0}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \tag{8}$$

in which $p_0 = p(T_0)$ is the saturated vapor pressure at temperature T_0, μ is the molar mass, and R_0 is the universal gas constant.

Substituting Eq. (8) into Eq. (6) with subsequent substitution of the particle number $N(t)$ into Eq. (5), we obtain for the internal gas energy in the cavity

$$U(t) = \frac{3}{2}V p_0 \exp\left[\frac{Q_r \mu}{R_0}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]. \tag{9}$$

The heat flux $q_T(t)$ through the cavity boundary, according to Eq. (4), is determined by the formula

$$q_T(t) = -\frac{Q_r R p_0 \mu}{2R_0 T^2}\exp\left[\frac{Q_r \mu}{R_0}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]\frac{dT(t)}{dt}. \tag{10}$$

For small changes in the temperature of the cavity ($|T(t) - T_0| << T_0$), the last expression can be represented in the form

$$q_T(t) = -\frac{Q_r R p_0 \mu}{2R_0 T_0^2} \frac{dT(t)}{dt}. \tag{11}$$

In its turn, the internal energy of the equilibrium photon gas in the cavity is determined by the equality

$$U(t) = \frac{4\sigma}{c} T^4(t) V, \tag{12}$$

where σ is the Stefan–Boltzmann constant and c is the velocity of light in vacuum. Then in the above-indicated approximation, we obtain for the heat flux $q_T(t)$ caused by the photon energy transfer:

$$q_T(t) = -\frac{16\sigma R T_0^3}{3c} \frac{dT(t)}{dt}. \tag{13}$$

Thus, the heat flux $q_T(t)$ through the cavity boundary caused by vapor particles and photon energy transfer is determined by the general expression

$$q_T(t) = -\frac{1}{A} \frac{dT(t)}{dt}, \tag{14}$$

where we have introduced the parameter

$$A = \frac{2R_0 T_0^2}{Q_r R p_0 \mu} \tag{15}$$

for the molecular gas and

$$A = \frac{3c}{16\sigma R T_0^3} \tag{16}$$

for the photon gas.

STOCHASTIC INTEGRAL EQUATION

In addition to the heat flux $q_T(t)$ caused by vapor condensation and liquid evaporation as well as by photon emission and absorption, the random heat flux $\xi_{q_T}(t)$ is always presented caused, for example, by local statistical fluctuations of the particle concentration or temperature of the medium. The character of the process $\xi_{q_T}(t)$ depends on concrete conditions and in most important cases, can be represented in the form of white noise with intensity ν. The noise spectrum is certainly limited by a frequency range with maximum frequency $\omega_{\max}$ and corresponding characteristic relaxation time τ_r of heat conduction process whose estimates are given by the expression

$$\tau_r = \frac{R^2}{\chi_n} \tag{17}$$

for heat flux fluctuations associated with vapor particle transfer and

$$\tau_r = \frac{R}{c} \tag{18}$$

for fluctuations caused by photon emission and absorption.

The intensity of fluctuations of heat flux ν can be estimated using the problem parameters. When the heat flux is transferred by vapor particles,

$$\nu = \frac{\kappa_n}{R^3} k_{\text{B}} T_0^2 \,, \tag{19}$$

and when it is transferred by photons,

$$\nu = \frac{\sigma}{R^2} k_{\text{B}} T_0^5 \,. \tag{20}$$

We now estimate the quantities entering into Eqs. (17)–(20) for the saturated water vapor ($\kappa_n = 0.02$ W/(K·m) and $\chi_n = 3.5 \cdot 10^{-4}$ m^2/s) enclosed in the cavity with radius $R = 1$ μm at the temperature $T_0 = 300$ K. For the molecular gas, we obtain $\tau_r = 2.9 \cdot 10^{-9}$ s and $\nu = 0.025$ J^2/(m^4·s), and for the photon gas in the cavity, $\tau_r = 3 \cdot 10^{-15}$ s and $\nu = 1.9 \cdot 10^{-6}$ J^2/(m^4·s). It can be easily seen that fluctuations caused by photon emission and absorption have much smaller characteristic time than fluctuations caused by evaporation and vapor condensation. This is due to a much higher velocity of photon motion in comparison with the velocity of thermal motion of vapor particles. The fluctuation intensities also differ considerably for the two above-indicated fluctuation types. This demonstrates the possibility of neglecting the photon component of the heat flux in the presence of the molecular component.

Equating the sum of flux (14) and random flux $\xi_{q_T}(t)$ to the general expression for the heat flux through the cavity surface

$$q_T(t) = -\kappa \left. \frac{\partial \tilde{T}(r,t)}{\partial r} \right|_{r=R} , \tag{21}$$

we obtain

$$-\kappa \left. \frac{\partial \tilde{T}(r,t)}{\partial r} \right|_{r=R} = -\frac{1}{A} \frac{dT(t)}{dt} - \xi_{q_T}(t) \,. \tag{22}$$

The sign of the random heat flux $\xi_{q_T}(t)$ in Eq. (22) is determined by the chosen positive flux direction. To find the derivative $\frac{\partial \tilde{T}(r,t)}{\partial r}$ in Eq. (21), we solve initial system of equations (1)–(3). For this purpose, we consider the auxiliary function $f(r,t)$ determined by the expression

$$f(r,t) = r\tilde{T}(r,t) \,. \tag{23}$$

Substitution of Eq. (23) into initial system (1)–(3) yields the corresponding system for the function $f(r,t)$:

$$\frac{\partial f(r.t)}{\partial t} = \chi \frac{\partial^2 f(r,t)}{\partial r^2} \,, \; r > R \,, \tag{24}$$

$$f(r,t)\big|_{t=0} = T_0 r\,, \tag{25}$$

$$f(r,t)\big|_{r=R} = RT(t)\,. \tag{26}$$

Equation (24) formally corresponds to the one-dimensional heat conduction equation with initial and boundary conditions (25) and (26). A solution of system (24)–(26) has two terms [5]. The first term is caused by the initial temperature distribution, and the second term is caused by the temperature changes on the boundary. From physical reasons, it is clear that the first term produces zero heat flux for the examined problem with allowance for the thermodynamic equilibrium at the initial moment of time. Taking this into account, only the second term remains in Eqs. (24)–(26). Thus,

$$f(r,t) = \frac{R}{2\sqrt{\pi\chi}} \int_0^t \frac{r-R}{(t-\tau)^{3/2}} \exp\left[-\frac{(r-R)^2}{4\chi(t-\tau)}\right] T(\tau)\,d\tau\,. \tag{27}$$

Let us find now the heat flux $q_T(t)$ using definition (21). We obtain

$$q_T(t) = -\kappa \frac{\partial \tilde{T}(r,t)}{\partial r}\bigg|_{r=R} = -\kappa \frac{\partial\left(f(r,t)/r\right)}{\partial r}\bigg|_{r=R} = -\frac{\kappa}{R}\left(\frac{\partial f(r,t)}{\partial t}\bigg|_{r=R} - T(t)\right). \tag{28}$$

Determining the derivative $\frac{\partial f(r,t)}{\partial r}$ from Eq. (27) and substituting it into formula (28), we finally write down

$$q_T(t) = \frac{\kappa}{\sqrt{\pi\chi}} \int_0^t \left(\frac{1}{\sqrt{t-\tau}} + \frac{\sqrt{\pi\chi}}{R}\right) \frac{dT(\tau)}{d\tau} d\tau\,. \tag{29}$$

Equating the relationship for the flux and the right side of Eq. (22), we obtain

$$\frac{\kappa}{\sqrt{\pi\chi}} \int_0^t \left(\frac{1}{\sqrt{t-\tau}} + \frac{\sqrt{\pi\chi}}{R}\right) \frac{dT(\tau)}{d\tau} d\tau = -\frac{1}{A}\frac{dT(t)}{dt} - \xi_{q_T}(t)\,. \tag{30}$$

Designating

$$Z(t) = \frac{dT(t)}{dt}\,, \tag{31}$$

we finally obtain the Volterra integral stochastic equation of the second kind for the function $Z(t)$:

$$Z(t) + \frac{A\kappa}{\sqrt{\pi\chi}} \int_0^t \left(\frac{1}{\sqrt{t-\tau}} + \frac{\sqrt{\pi\chi}}{R}\right) Z(\tau)\,d\tau = -A\xi_{q_T}(t)\,. \tag{32}$$

It should be emphasized that integral equation (32) cannot be reduced to final system of stochastic differential equations, and consequently, the process $Z(t)$ together with processes $T(t)$ and $q_T(t)$ related to it is a non-Markov random process [6]. We note that problems of spherical Brownian particle motion in an infinite viscous medium and of heat transfer in the space surrounding the particle made of the material with high heat conduction [3] are reduced to equations analogous to Eq. (2) [2].

A solution of Eq. (32) can be generally represented in the form of the Volterra integral operator of the second kind [7]

$$Z(t) = -A\xi_{q_T}(t) - \int_0^t K(t-\tau)\xi_{q_T}(\tau)\,d\tau, \tag{33}$$

in which the function $K(t-\tau)$ is a resolvent for Eq. (30) represented by a series

$$K(t-\tau) = \sum_{n=1}^{\infty} u_n(t-\tau), \tag{34}$$

and

$$u_n(t-\tau) = -\int_\tau^t u_1(t-s)u_{n-1}(s-\tau)\,ds,\ u_1(t-\tau) = \frac{A\kappa}{\sqrt{\pi\chi}}\left(\frac{1}{\sqrt{t-\tau}} + \frac{\sqrt{\pi\chi}}{R}\right). \tag{35}$$

Equation (32) can also be solved analytically. Indeed, let us take the Laplace transform of initial equation (32). Considering the function

$$\delta T(t) = T(t) - T_0 \tag{36}$$

that determines the difference between the current temperature in the cavity and the initial temperature T_0, from Eq. (32) we obtain for the transform $\delta\widehat{T}(s)$:

$$\delta\widehat{T}(s) = -\frac{A\widehat{\xi}_{q_T}(s)}{s + \frac{\beta\kappa}{\sqrt{\chi}}\sqrt{s} + \frac{\beta\kappa}{R}}, \tag{37}$$

where $\widehat{\xi}_{q_T}(s)$ is the transform of the function $\xi_{q_T}(s)$. Factorizing denominator of Eq. (37), we obtain

$$\delta\widehat{T}(s) = -\frac{A\widehat{\xi}(s)}{s_0}\left(\frac{1}{\sqrt{s}-s_1} - \frac{1}{\sqrt{s}-s_2}\right), \tag{38}$$

where

$$s_0 = \sqrt{\frac{A^2\kappa^2}{\chi} - \frac{4A\kappa}{R}},\ s_1 = -\frac{A\kappa}{2\sqrt{\chi}} + \frac{s_0}{2},\ s_2 = -\frac{A\kappa}{2\sqrt{\chi}} - \frac{s_0}{2}. \tag{39}$$

From Eq. (38), we easily find

$$\delta T(t) = -\frac{A}{s_0}\int_0^t K(t-\tau)\xi_{q_T}(\tau)\,d\tau, \tag{40}$$

where the kernel $K(t-\tau)$ is the inverse Laplace transform of the function in the parentheses in the right side of Eq. (38) [8]:

$$K(t-\tau) = s_1\exp\left(s_1^2(t-\tau)\right)\operatorname{erfc}\left(-s_1\sqrt{t-\tau}\right) - s_2\exp\left(s_2^2(t-\tau)\right)\operatorname{erfc}\left(-s_2\sqrt{t-\tau}\right). \tag{41}$$

Here $\mathrm{erfc}(x)$ is the additional integral of errors defined by the expression [9]

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp\left(-z^2\right) dz \,. \tag{42}$$

For large t (a steady-state process), we expand the function $\mathrm{erfc}(x)$ in a series [9] considering only the first two terms and using Eq. (39). Then the kernel $K(t-\tau)$ is reduced to the form

$$K(t-\tau) = \frac{s_0 R^2}{2A\kappa\sqrt{\pi\chi}} \frac{1}{(t-\tau)^{3/2}} \,. \tag{43}$$

Substituting the formula obtained into Eq. (40), we finally obtain the following expression for the gas temperature fluctuations in the cavity:

$$\delta T(t) = -\frac{R^2}{2\kappa\sqrt{\pi\chi}} \int_0^t \frac{1}{(t-\tau)^{3/2}} \xi_{q_T}(\tau)\, d\tau \,. \tag{44}$$

We note that the parameter A depending on the gas properties is not included in the formula for temperature fluctuations of the steady-state process. Thus, the character of steady-state fluctuations in the examined linear approximation is determined only by the properties of the medium surrounding the cavity, cavity sizes, and equilibrium temperature and is independent of the gas state equation in the cavity. This fact, in particular, demonstrates the applicability of the examined method for temperatures at which the molecular gas does not obey the ideal gas equation, for example, near the critical temperature.

STATISTICAL CHARACTERISTICS OF TEMPERATURE FLUCTUATIONS

Let us find the statistical characteristics of random fluctuations of the temperature difference $\delta T(t)$ between the gas temperature in the cavity and the equilibrium temperature taking advantage of Eqs. (40) and (44). For the one-dimensional characteristic function $g_1(\lambda;t)$, from Eq. (40) we obtain the expression [10]

$$g_1(\lambda;t) = \exp\left[-\frac{1}{2}\nu\lambda^2 \frac{A^2}{s_0^2} \int_0^t K^2(t-\tau)\, d\tau\right] \tag{45}$$

in which the intensity of fluctuations of the heat flux ν are determined by formulas (19) and (20) and the kernel $K(t-\tau)$ is given by Eq. (41).

According to Eq. (44), for a steady-state process we have the one-dimensional characteristic function of the form

$$g_1(\lambda;t) = \exp\left[-\frac{\nu R^4 \lambda^2}{16\pi\kappa^2\chi}\left(\frac{1}{\tau_r^2} - \frac{1}{t^2}\right)\right] \tag{46}$$

obtained after replacement of the upper integration limit in Eq. (45) by $t-\tau_r$, which is caused by the absence of physical influence on the process at the present time of preceding states of the process at time τ_r. As follows from Eq. (46), the one-dimensional characteristic function is independent of the gas properties in the cavity and its equation of state for $t \gg \tau_r$; it is determined by the intensity of fluctuations of ν and characteristic time τ_r. The characteristic

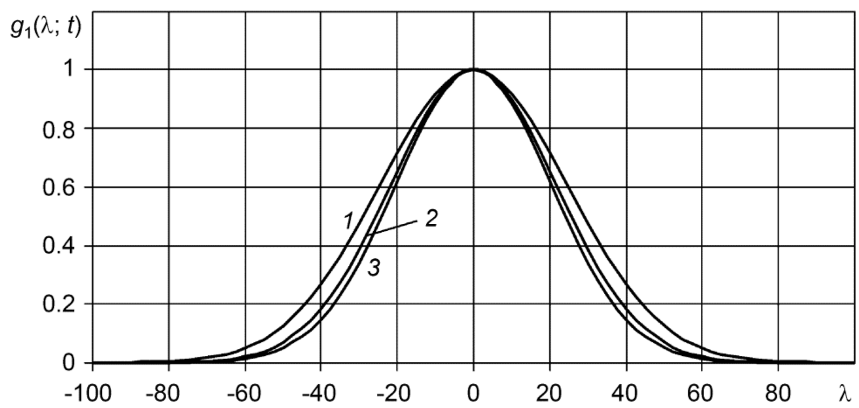

Fig. 2. Characteristic functions $g_1(\lambda;t)$ described by Eq. (46) at $t=2\tau_r$ (curve *1*), $3\tau_r$ (curve *2*), and ∞ (curve *3*).

functions for the above-considered micron cavity in water described by Eq. (46) are shown in Fig. 2 for the indicated times *t*.

The one-dimensional characteristic function $g_1(\lambda;t)$ so obtained can be used to determine moments of the process $\delta T(t)$ of any arbitrary order. For the mathematical expectation $\langle \delta T(t)\rangle$, we obtain

$$\langle \delta T(t)\rangle = \left.\frac{\partial g_1(\lambda;t)}{\partial (i\lambda)}\right|_{\lambda=0} = 0\,, \tag{47}$$

and for the variance $\langle \delta T^2(t)\rangle$, we have

$$\langle \delta T^2(t)\rangle = \left.\frac{\partial^2 g_1(\lambda;t)}{\partial (i\lambda)^2}\right|_{\lambda=0} = \nu\frac{A^2}{s_0^2}\int_0^t K^2(t-\tau)d\tau\,, \tag{48}$$

which for a steady-state process is reduced to the expression

$$\langle \delta T^2(t)\rangle = \frac{R^4\nu}{8\pi\kappa^2\chi}\left(\frac{1}{\tau_r^2}-\frac{1}{t^2}\right). \tag{49}$$

We note that when $t\to\infty$, the variance of $\delta T(t)$ fluctuations tends to a constant

$$\left.\langle \delta T^2(t)\rangle\right|_{t\to\infty} = \frac{R^4\nu}{8\pi\kappa^2\chi\tau_r^2}\,. \tag{50}$$

The one-dimensional distribution density function $f_1(\delta T;t)$ for fluctuations of temperature changes $\delta T(t)$ can be found from Eqs. (45) and (46) using the definition [1]

$$f_1(\delta T;t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g_1(\lambda;t)\exp(-i\lambda\delta T)d\lambda\,. \tag{51}$$

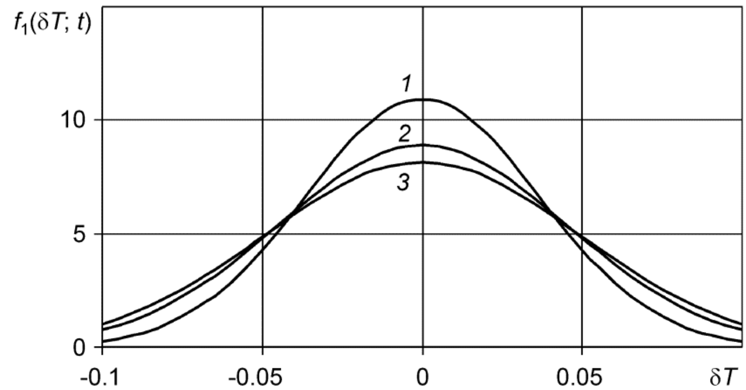

Fig. 3. Plots of the distribution density function $f_1(\delta T;t)$ determined by Eq. (52) at $t = 2\tau_r$ (curve *1*), $3\tau_r$ (curve *2*), and ∞ (curve *3*).

In particular, for a steady-state process, after substitution of Eq. (49) into formula (46), we obtain

$$f_1(\delta T;t) = \frac{1}{\sqrt{2\pi\left\langle \delta T^2(t)\right\rangle}} \exp\left[-\frac{\delta T^2}{2\left\langle \delta T^2(t)\right\rangle}\right]. \tag{52}$$

Thus, the one-dimensional distribution function at each moment of time is a Gaussian process; moreover, with increasing *t*, the function $f_1(\delta T;t)$ spreads along the temperature axis (Fig. 3).

Let us now find the spectral power density $G_{\delta T}(\omega)$ of steady-state fluctuations of function $\delta T(t)$. From Eq. (37) we obtain

$$G_{\delta T}(\omega) = \frac{A^2\nu}{\omega^2 + \frac{A\kappa}{\sqrt{\chi}}\sqrt{2\omega^3} + \frac{A^2\kappa^2}{\chi}\omega + \frac{A^2\kappa^2}{R\sqrt{\chi}}\sqrt{2\omega} + \frac{A^2\kappa^2}{R^2}}. \tag{53}$$

In particular, for $\omega \to 0$, the spectral density $G_{\delta T}(\omega)$ tends to a constant

$$G_{\delta T}(\omega)\Big|_{\omega\to 0} = \frac{\nu R^2}{\kappa^2}. \tag{54}$$

Figures 4 and 5 show the spectral density $G_{\delta T}(\omega)$ described by formula (53) for the indicated cavity radii and molecular and photon gases, respectively. It can be easily seen that the intensities of temperature fluctuations of the molecular and photon gases decrease with increasing cavity radius. We note also that for small ω values, the spectral density of photon gas temperature fluctuations is independent of the cavity sizes. This is caused by inverse proportionality of the intensity of fluctuations of the random heat flux component to the squared cavity radius.

CONCLUSIONS

Thus, the heat conduction process in the medium comprising microcavities studied in the present work calls for the application of the integral stochastic equations for a description of fluctuations of the corresponding physical quantities, and in this case, the fluctuations themselves are non-Markov random processes. The character of the steady-

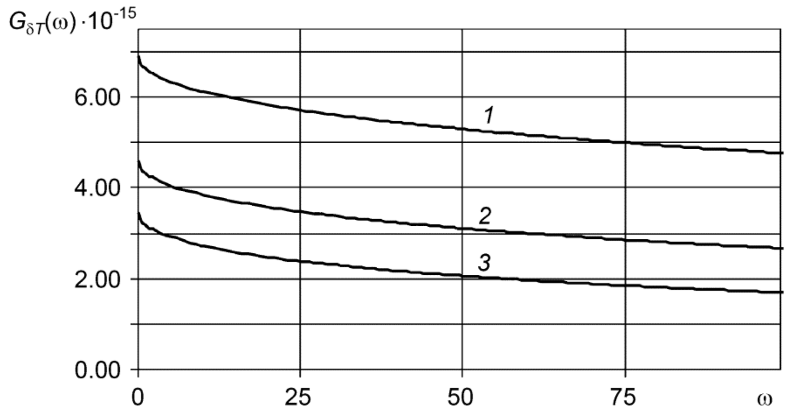

Fig. 4. Plots of the spectral density $G_{\delta T}(\omega)$ determined by formula (53) for the molecular gas at $R = 10$ (curve *1*), 15 (curve *2*), and 20 μm (curve *3*).

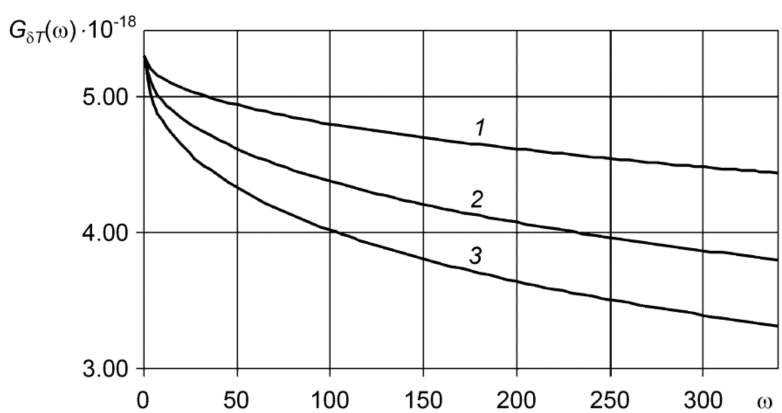

Fig. 5. Plots of the spectral density $G_{\delta T}(\omega)$ determined by formula (53) for the photon gas at $R = 1$ (curve *1*), 2 (curve *2*), and 3 μm (curve *3*).

state heat conduction process in the linear approximation appears independent of the state equation for the gas enclosed in the cavity. The results obtained can be used to describe random temperature fluctuations in media with a microstructure.

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