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Processing of Double-Sided Interferograms Subject to Background Self-Radiation of an FTIR Spectrometer

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Abstract—The effect of background (instrumental) self-radiation of a Fourier transform infrared (FTIR) spectrometer on the processing of double-sided interferograms is studied. From a theoretical analysis of the Michelson interferometer it was shown that when the brightness temperature of the object of study is close to the temperature of the instrument, the measured spectrum may contain inverted spectral ranges or isolated bands. The interferogram correction algorithms and experimental technique for elimination of the spectrum inversion and for compensation of the background self-radiation of the instrument are suggested. © *2002 MAIK "Nauka/Interperiodica"*.

INTRODUCTION

Consideration of background (instrumental) radiation is an important problem in almost every branch of experimental physics. In most cases, a background signal is additively combined to true measured signal. There exist many approved procedures and experimental techniques which make it possible to readily distinguish between true and background signals. In some other cases, however, this distinction is not so evident and specially developed procedures are required to account for the background component of the signal. This is especially true in Fourier spectroscopy, more specifically, in Fourier radiometry, where the input signal from the object under study is comparable in power with the background instrumental self-radiation. In this case, the directly measured initial interferogram may contain phase errors due to the Narcissus effect, which cannot be removed by using the standard processing procedures employed in laboratory instruments [1, 2].

The objective of this study was to develop and approve a technique and correction algorithm for compensation of the background self-radiation of instruments in double-sided interferograms of FTIR spectrometers used for the observation of natural underlying surfaces.

STATEMENT OF THE PROBLEM

We will demonstrate the essence of the problem by the following experimental fact. Figure 1 (curve *1*) shows the radiation spectrum of clear sky measured at an angle of 60° to the horizon plane with a Fourier spectroradiometer at a resolution of ~4 cm^{-1} [3, 4]. Below, a reference spectrum (curve *2*) is shown that exhibits the proper positions and intensities of the spectral bands. It can be clearly seen that both isolated bands and entire spectral ranges are inverted in the upper spectrum. It is evident that formal application of a routine spectrum restoration procedure leads to erroneous results. This clearly indicates a need for special correction methods.

In order to determine the causes of the above effect, let us consider the routine (used to obtain the spectrum *1* in Fig. 1) procedure of interferogram processing. Assuming phase distortion to be absent, the following relationships arise:

$$I(x) = 2\pi\int_{-\infty}^{\infty} B_0(\nu)\exp(2\pi i\nu x)d\nu,$$
$$B_0(\nu) = \int_{-\infty}^{\infty} I(x)\exp(-2\pi i\nu x)dx, \quad (1)$$

where $I(x)$ is the recorded signal of the interferogram at the arm unbalance of interferometer x and $B_0(\nu)$ is the restored radiation spectrum of the object under study at the frequency ν (cm^{-1}).

In relationship (1), the limits of integration in actual instruments are finite and the interferogram is sampled discretely; however, these circumstances exert no effect on what is considered further. The functions $I(x)$ and $B_0(\nu)$ in relationships (1) are real and even (symmetrical). Let the amplitude function $R_0(\nu)$ account for the sensitivity of the detector and the phase function $\varphi(\nu)$ take into consideration the phase distortion of the instrument. Then, instead of the first relationship in (1) we actually have

$$I(x) = 2\pi\int_{-\infty}^{\infty} R_0(\nu)B_0(\nu)\exp[i(2\pi\nu x - \varphi(\nu))]d\nu,$$

and the result of the spectrum restoration becomes complex:

$$\tilde{B}(\nu) = R_0(\nu)B_0(\nu)\exp(-i\varphi(\nu)). \quad (2)$$

When processing one-sided interferograms, the presence of phase distortion requires preliminary phase compensation. Double-sided interferograms do not usually need such a procedure, because the equality

$$B_0(\nu) = \frac{|\tilde{B}(\nu)|}{R_0(\nu)}, \quad (3)$$

following from (2), holds true for positive values of the function $B_0(\nu)$. In this case, the function $R_0(\nu)$ can be determined beforehand from the spectrum of a standard IR source with known radiative properties. However, if the background self-radiation of the instrument is taken into account, the function $B_0(\nu)$ may become alternating. In this case, the simple relationship (3) cannot be used for restoration of the spectrum from the double-sided interferograms.

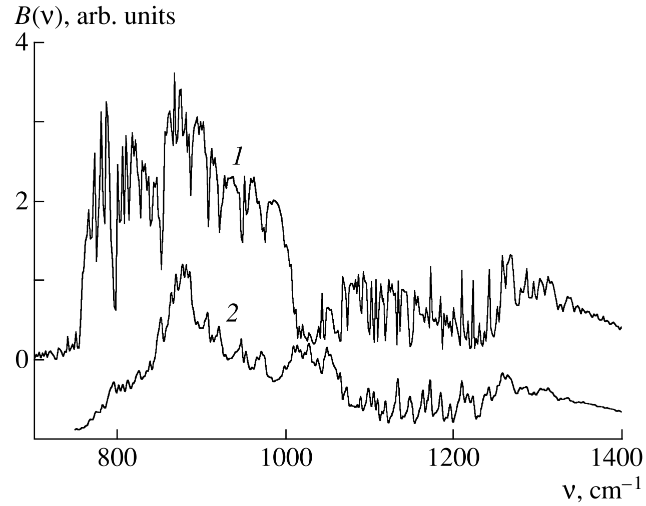

Fig. 1. (*1*) Radiation spectrum of clear sky measured at an angle of 60° to the horizon plane using a FTIR spectrometer with a resolution of ~4 cm^{-1}, (*2*) reference spectrum.

THE EFFECT OF BACKGROUND SELF-RADIATION OF A FTIR SPECTROMETER

Let us consider radiation incident on a photodetecting device (PDD) of an ideal Michelson interferometer (at $R_0 = 1$, $\varphi(\nu) = 0$, and with the coefficient of splitting of the light beam equal to 0.5) subject to a spectrum of background self-radiation. It was shown in [5] that the interferogram of radiation at a frequency ν from an object under study whose spectral radiating power alone is B_0 can be expressed as

$$I_1(x, \nu) = \frac{B_0}{2}(1 + \cos(2\pi\nu x)),$$

where x is the absolute coordinate of displacement of a movable mirror with respect to a zero point. From the law of conservation of energy we can obtain the following expression for the reflected (backward) flux:

$$I_2(x, \nu) = \frac{B_0}{2}(1 - \cos(2\pi\nu x)).$$

Similar reasoning can be used for the background self-radiation of the interferometer $F_0(\nu)$ that enters the instrument from the direction of the photodetecting device.

Physically, the background flux forms due to multiple reflection of the IR radiation from basic parts of the instrument by coated surfaces of the collecting lens system. Another source of the background flux is the scattering of IR radiation by the aperture of the photodetector when the spherical aberration of the collecting lens system is considerable, which is the case if single-lens short-focus objectives are used. Some contribution may also arise from the background self-radiation of various optical components of the instrument, such as the mirrors, beam splitter, etc. As a result, we can obtain the following expression for the total signal of the interferogram at the PDD (the constant component equal $(B_0 + F_0)/2$ is ignored):

$$I(x, \nu) = \frac{B_0 - F_0}{2}\cos(2\pi\nu x).$$

Therefore, the interferogram is formed by a difference signal, which, allowing for the amplitude function $R_0(\nu)$ and function of phase distortion $\varphi(\nu)$, leads to the following modification of formula (2):

$$\tilde{B}(\nu) = R_0(\nu)(B_0(\nu) - F_0(\nu))\exp(-i\varphi(\nu)), \quad (4)$$

where $\tilde{B}(\nu)$ is the measured spectrum and $B_0(\nu)$ and $F_0(\nu)$ describe the densities of energy distribution in the spectra of the object under study and of the FSR self-radiation.

If the relation $B_0(\nu) \geq F_0(\nu)$ is valid in the entire frequency range of interest, then the following analogue of relation (3) holds true

$$B(\nu) \equiv |\tilde{B}(\nu)| = R_0(\nu)[B_0(\nu) - F_0(\nu)]. \quad (5)$$

Experimentally, in order to find the functions $R_0(\nu)$ and $F_0(\nu)$, it is sufficient to measure two reference spectra of two standard sources with known radiative properties and maximally different brightness temperatures. Indeed, having two experimentally determined spectra B_1 and B_2 of both IR sources with known initial spectra ($B_{01} = P_1$ and $B_{02} = P_2$, correspondingly), one can

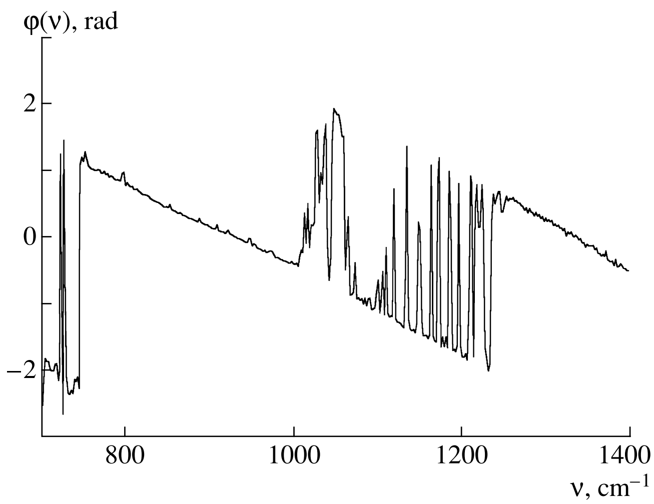

Fig. 2. Function of phase distortion of the radiation spectrum of clear sky.

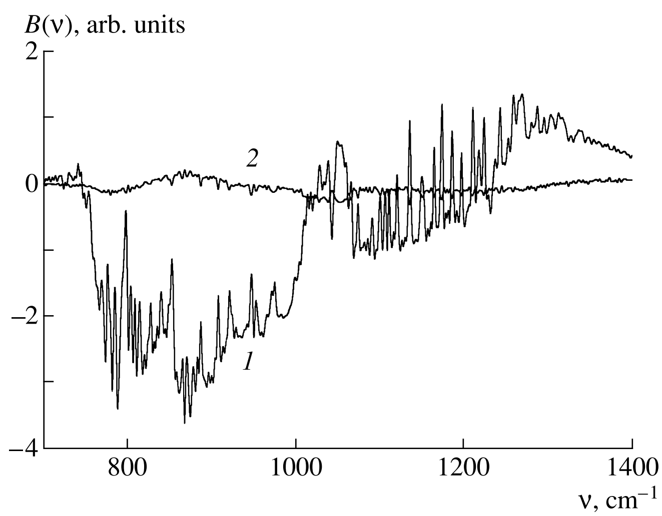

Fig. 3. (*1*) Real and (*2*) imaginary parts of the experimental spectrum subjected to the correction of phase distortion.

readily express the desired functions from (5) algebraically:

$$\left.\begin{aligned} R_0 &= \frac{B_2 - B_1}{P_2 - P_1} \\ F_0 &= \frac{P_1 B_2 - P_2 B_1}{B_2 - B_1}. \end{aligned}\right\} \quad (6)$$

Notice that the absolute values $B_1(\nu)$ and $B_2(\nu)$ of complex spectra appear in Eqs. (6). Commonly, this correction procedure is used in FTIR spectrometers with uncooled PDDs to determine the absolute value of the spectral density of radiance from the objects under study [6, 7].

PHASE CORRECTION ALGORITHM

Processing of experimental interferograms becomes considerably complicated if the difference signal of incident radiation $B_0(\nu) - F_0(\nu)$ changes sign within the spectral range of interest. In this case, only the absolute value of the difference rather than its sign can be determined from relations similar to (5). It can be seen from formula (4) that the difference signal can be determined from the measured complex spectrum, if the function of phase distortion $\varphi(\nu)$ is known. In practice, this function is unknown and, moreover, may vary from measurement to measurement due to, for example, inaccuracy in determining the zero point of an asymmetric interferogram. The phase can be determined directly from relation (4) as the argument of the complex spectrum, but this determination is ambiguous: both the change of sign of the difference signal and change of the phase function by π may lead to the same result.

This ambiguity can be removed provided the function of phase distortion is a smooth slowly varying function of frequency. Thus, the phase of the spectrum (4) should be a smooth slowly varying function of frequency, abruptly changing by π at every point where the difference signal $B_0(\nu) - F_0(\nu)$ changes sign. It is this behavior of the phase that can be seen in Fig. 2, with the phase steps not always being exactly equal to π because of the limited resolution of the instrument and noise interference.

As a result, the function of phase correction can be obtained by separating the continuous component of the phase from the measured complex spectrum. Technically, this can be done by approximating the function $\varphi(\nu)$ with a polynomial of low degree, for example, the quadratic $\varphi(\nu) = a + b\nu + c\nu^2$, whose coefficients are determined from the minimum of the squared imaginary part

$$B(\nu) = \tilde{B}(\nu)\exp(i\varphi(\nu)), \quad (7)$$

i.e., the function

$$\Phi(a, b, c) = \int_{\nu_{\min}}^{\nu_{\max}} [\mathrm{Re}\,\tilde{B}(\nu)\sin(\varphi(\nu)) + \mathrm{Im}\,\tilde{B}(\nu)\cos(\varphi(\nu))]^2 d\nu,$$

where $\nu_{\min}$ and $\nu_{\max}$ are the minimal and maximal frequencies of the measured spectrum, respectively. Actual calculations show that the proposed model for the description of phase incursion is adequate, because it is possible to obtain spectral function (7) with a sufficiently small imaginary part (Fig. 3). This procedure is somewhat similar to that used for the phase correction of one-sided interferograms, where the phase function is determined from a small portion of the double-sided interferogram and thus automatically turns out to be sufficiently smooth.

Once the function of phase distortion is determined, the algorithm of the correction to the spectrum is reduced to the addition of the function of self-radiation of the instrument $F_0(\nu)$. The compensation corrections in this case can be found using the same formulas (6), into which the values calculated from the initial spectra in accordance with (7) should be substituted instead of B_1 and B_2.

DETERMINATION OF THE BACKGROUND SPECTRAL CHARACTERISTICS OF A FTIR SPECTROMETER

The characteristics of the instrument derived from (6) need further specification since the self-radiation of the instrument $F_0(\nu)$ depends on temperature, which varies during the operation of the instrument. Assuming that the temperature of various parts of the interferometer T_p is spatially uniform, formula (4) subject to relation (7) can be rewritten as

$$B(\nu) = R_0(\nu)[B_0(\nu) - \varepsilon(\nu)P(\nu, T_p)],$$

where $P(\nu, T_p)$ is the Planck's function for the temperature of the FTIR spectrometer and $\varepsilon(\nu)$ is the emittance of the background source, which is considered to be independent of temperature. Therefore, in order to remove the background component from the initial spectra, the two functions $R_0(\nu)$ and $\varepsilon(\nu)$ must be known. They can be found from measurements of spectra from standard IR sources maintained at different temperatures provided the interferometer temperature of the FTIR spectrometer is varied. Let there be several measurements of spectra $B_i(\nu)$ carried out for IR sources with emittances T_i maintained at temperatures ε_i ($i = 1, \ldots, n$). During each measurement, the housing of the interferometer is maintained at the temperature T_{pi} by means of a control device. Then, we have the following set of equations to determine the functions $R_0(\nu)$ and $\varepsilon(\nu)$:

$$B_i(\nu) = R_0(\nu)[\varepsilon_i P(\nu, T_i) - \varepsilon(\nu)P(\nu, T_{pi})] = R_0\varepsilon_i P(T_i) - \beta P(T_{pi}),$$

where a new function $\beta = R_0\varepsilon$ is introduced instead of ε. For each frequency ν we thus have a set of n linear equations with respect to the quantities R_0 and β. Considering that this set is overdetermined for $n > 2$, we

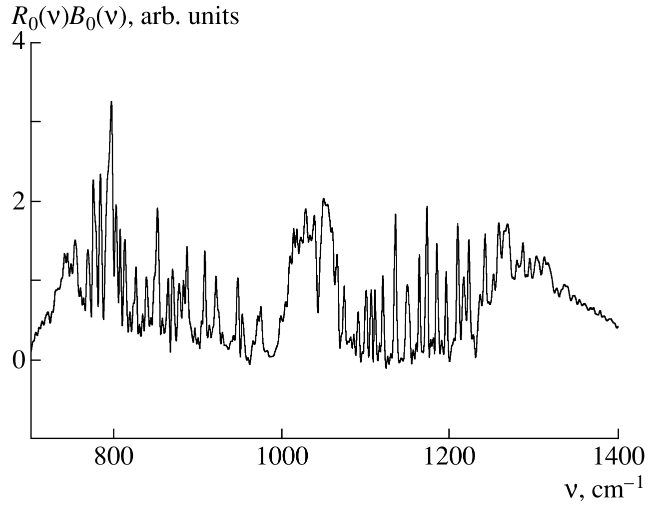

Fig. 4. Radiation spectrum of clear sky subjected to the correction of phase distortion and compensation of the FTIR spectrometer background self-radiation.

may obtain the following equations to determine R_0 and β for each frequency value ν:

$$\left.\begin{aligned} \left[\sum_{i=1}^{n}\varepsilon_i^2 P^2(T_i)\right]R_0 - \left[\sum_{i=1}^{n}\varepsilon_i P(T_i)P(T_{pi})\right]\beta &= \sum_{i=1}^{n}\varepsilon_i P(T_i)B_i \\ \left[\sum_{i=1}^{n}\varepsilon_i P(T_i)P(T_{pi})\right]R_0 - \left[\sum_{i=1}^{n}P^2(T_{pi})\right]\beta &= \sum_{i=1}^{n}P(T_{pi})B_i. \end{aligned}\right\} \quad (8)$$

By solving set (8) for each ν, we can find the desired functions $R_0(\nu)$ and $\varepsilon(\nu) = \beta(\nu)/R_0(\nu)$. We note that in order to improve the conditionality of set (8), the range of temperature variation of the instrument and IR sources should be the maximum possible but should not exceed the range of linearity of the photodetecting device.

AN EXAMPLE OF SPECTRUM RESTORATION

We will demonstrate the procedure of restoration of the spectrum with incident-radiation inversion using the following example. Figure 1 (curve *1*) shows the amplitude of an experimentally observed spectrum of clear sky measured at an elevation angle of 60°, and Fig. 2 presents the phase of this spectrum. It can be seen from the amplitude spectrum that the atmospheric lines (bands) are inverted over a large spectral range and, from the phase spectrum, that the phase along the expected continuous change undergoes abrupt changes of about π. The correction of the phase in this case leads to significant changes and is necessary for correct interpretation and further processing of the spectrum.

In order to construct the background spectral characteristics of the FTIR spectrometer, we measured several spectra of IR sources ($\varepsilon = 0.96–0.98$) maintained at temperatures from 0 to 39°C and with the interferometer temperature varying from 16 to 32°C. As was expected, the emittance values $\varepsilon(\nu)$ were positive and less than unity over the entire spectral range of interest, with the function $R_0(\nu)$ also being positive. The true difference spectrum restored by the phase correction is shown in Fig. 3. It can be seen that in the frequency range from 780 to 1050 cm^{-1} the self-radiation of the instrument exceeds the radiation from the sky, resulting in negative values of the difference spectrum $B(\nu)$. The compensation of the background self-radiation leads to a nonnegative final spectrum $R_0(\nu)B_0(\nu)$ (Fig. 4) in which inverted regions are entirely absent. Comparison of the experimental spectra with those calculated using the proposed model showed an insignificant discrepancy, “subsidence” of the treated spectrum with respect to the calculated one. We believe that this discrepancy could result from the poor quality of the used IR sources and from the low measurement accuracy of their emittances, as well as the actual temperature and its stability over time. Some error may result from the nonuniformity in the temperature field of the interferometer parts due to their nonuniform heating.

CONCLUSION

The experimental technique and correction algorithm for the initial double-sided interferograms of the FTIR spectrometer are proposed and approved. Their application makes it possible to avoid inversion in the treated spectra caused by background self-radiation of the FTIR spectrometer when performing field observations.

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